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A REFINED THEORY OF ANISOTROPIC SHELLS

by

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13. ABSTRACT

Construction of a new refined theory of anisotropic shells not involving the widely held assumption that the normal displacement is independent of the coordinate of the normal to the middle surface. It is shown that, in contrast to all other refined theories of the class considered, the proposed geometrical model of shell deformation is such that all the displacement components at a given point on the shell depend nonlinearly on the above mentioned coordinate.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

A REFINED THEORY OF ANISOTROPIC SHELLS

S. A. Ambartsumyan (Yerevan)

1. In this work is constructed a new refined theory of anisotropic shells which is based on the following assumptions:

a) when defining deformations $e_{\alpha\gamma}$ and $e_{\beta\gamma}$ is considered that shearing stresses $\tau_{\alpha\gamma}$ and $\tau_{\beta\gamma}$ are not distinguished from the appropriate stresses $\tau_{\alpha\gamma}{}^{\circ}$ and $\tau_{\beta\gamma}{}^{\circ}$, found from the classical theory;

b) deformation e_{γ} and normal stress σ_{γ} are not distinguished from the appropriate values ($e_{\gamma}{}^{\circ}$, $\sigma_{\gamma}{}^{\circ}$) of the classical theory.

As usual, the shell is related to a triorthogonal system of curvilinear coordinates α, β, γ, where γ is the rectilinear coordinate normal to the middle surface of the shell. At every point of the shell there is only one plane of elastic symmetry, parallel to the middle surface. The displacements of the shell are small, while deformations are subject to the generalized Hooke's law for an anisotropic body. The accepted here unspecified designations have been taken from works [1, 2].

The distinction of the proposed theory from earlier known theories is here there is no widespread hypothesis about the

independence of normal displacement u_γ on coordinate γ.

Taking hypotheses a) and b), which are actually approximately, it is assumed that

$$\tau_{\alpha\gamma} = \tau_{\alpha\gamma}^{\circ}, \quad \tau_{\beta\gamma} = \tau_{\beta\gamma}^{\circ}, \quad \sigma_\gamma = \sigma_\gamma^{\circ}; \quad e_{\alpha\gamma} = e_{\alpha\gamma}^{\circ}, \quad e_{\beta\gamma} = e_{\beta\gamma}^{\circ}, \quad e_\gamma = e_\gamma^{\circ} \tag{1.1}$$

Finally, let us say that if we treat the classical theory as the zero approximation, the proposed theory can be considered as the subsequent first approximation.

2. From the classical theory [1] for stresses $\tau_{\alpha\gamma}{}^{\circ}$, $\tau_{\beta\gamma}{}^{\circ}$, and $\sigma_\gamma{}^{\circ}$ we have (Fig. 1).

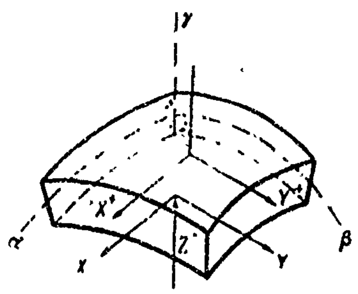

Fig. 1.

$$\begin{aligned}
\tau_{\alpha\gamma}^{\circ} &= X_1 + \frac{\gamma}{h} X_2 + \frac{1}{2}\left(\frac{h^2}{4} - \gamma^2\right)\varphi_0 \\
\tau_{\beta\gamma}^{\circ} &= Y_1 + \frac{\gamma}{h} Y_2 + \frac{1}{2}\left(\frac{h^2}{4} - \gamma^2\right)\psi_0 \\
\sigma_\gamma{}^{\circ} &= Z_1 + \frac{\gamma}{h} Z_2 - \frac{\gamma}{6}\left(\frac{h^2}{4} - \gamma^2\right)\frac{1}{AB}\left[(B\varphi_0)_{,\alpha} + (A\psi_0)_{,\beta}\right] - \\
&\quad - \frac{6}{h^3}\left(\frac{h^2}{4} - \gamma^2\right)(k_1 M_1{}^{\circ} + k_2 M_2{}^{\circ})
\end{aligned} \tag{2.1}$$

In expressions (2.1) the following designations are used:

$$\begin{gathered}
X_1 = {}^1/_2 (X^+ - X^-), \quad Y_1 = {}^1/_2 (Y^+ - Y^-), \quad Z_1 = {}^1/_2 (Z^+ - Z^-) \\
X_2 = X^+ + X^-, \quad Y_2 = Y^+ + Y^-, \quad Z_2 = Z^+ + Z^-
\end{gathered} \tag{2.2}$$

$$\begin{aligned}
\varphi_0 &= -\frac{1}{AB}\{[AI_2(B_{26})]_{,\beta} + A_{,\beta} I_2(B_{66}) + [BI_2(B_{11})]_{,\alpha} - B_{,\alpha} I_2(B_{22})\} w_0 \\
\psi_0 &= -\frac{1}{AB}\{[BI_2(B_{26})]_{,\alpha} + B_{,\alpha} I_2(B_{66}) + [AI_2(B_{22})]_{,\beta} - A_{,\beta} I_2(B_{11})\} w_0
\end{aligned} \tag{2.3}$$

$$l_2(B_{ik}) = \frac{B_{2k}}{B}\left[\frac{\partial}{\partial\beta}\left(\frac{1}{B}\frac{\partial}{\partial\beta}\right) + \frac{1}{A^2}\frac{\partial B}{\partial\alpha}\frac{\partial}{\partial\alpha}\right] + 2\frac{B_{6k}}{AB}\left[\frac{\partial^2}{\partial\alpha\,\partial\beta} - \frac{1}{B}\frac{\partial B}{\partial\alpha}\frac{\partial}{\partial\beta} - \right.$$

$$\left. - \frac{1}{A}\frac{\partial A}{\partial\beta}\frac{\partial}{\partial\alpha}\right] + \frac{B_{1k}}{A}\left[\frac{\partial}{\partial\alpha}\left(\frac{1}{A}\frac{\partial}{\partial\alpha}\right) + \frac{1}{B^2}\frac{\partial A}{\partial\beta}\frac{\partial}{\partial\beta}\right] \qquad (2.4)$$

$$M_1^\circ = D_{11}\varkappa_1^\circ + D_{12}\varkappa_2^\circ + D_{16}\tau^\circ, \quad H^\circ = D_{16}\varkappa_1^\circ + D_{26}\varkappa_2^\circ + D_{66}\tau^\circ$$

$$M_2^\circ = D_{22}\varkappa_2^\circ + D_{12}\varkappa_1^\circ + D_{26}\tau^\circ, \quad D_{ik} = {}^1/_{12}\, h^3 B_{ik} \qquad (2.5)$$

Here B_{ik} – elastic constants; M_i°, H° – internal moments, h – the thickness of the shell, D_{ik} – the stiffness of the bend, κ_i°, τ° – the changes in curvature, and twisting.

It is evident that all magnitudes with subscripts zero represent the classical theory.

Further, from the classical theory, according to the generalized Hooke's law, for deformations we have

$$e_{\alpha\gamma}^\circ = X^* + \frac{\gamma}{h}X' + \frac{1}{2}\left(\frac{h^2}{4} - \gamma^2\right)\Phi_1^\circ$$

$$e_{\beta\gamma}^\circ = Y^* + \frac{\gamma}{h}Y' + \frac{1}{2}\left(\frac{h^2}{4} - \gamma^2\right)\Phi_2^\circ \qquad (2.6)$$

$$e_\gamma^\circ = \frac{1}{h}(a_{13}T_1^\circ + a_{23}T_2^\circ + a_{36}S^\circ) + \gamma\frac{12}{h^3}(a_{13}M_1^\circ + a_{23}M_2^\circ + a_{36}H^\circ) -$$

$$- \frac{\gamma}{6}\left(\frac{h^2}{4} - \gamma^2\right)\frac{a_{33}}{AB}\left[(B\varphi_0)_{,\alpha} + (A\psi_0)_{,\beta}\right] -$$

$$- \frac{6}{h^3}\left(\frac{h^2}{4} - \gamma^2\right)a_{33}(k_1M_1^\circ + k_2M_2^\circ) + a_{33}\left(Z_1 + \frac{\gamma}{h}Z_2\right) \qquad (2.7)$$

where

$$X^* = a_{55}X_1 + a_{45}Y_1, \quad X' = a_{55}X_2 + a_{45}Y_2, \quad \Phi_1^\circ = a_{55}\varphi_0 + a_{45}\psi_0$$

$$Y^* = a_{44}Y_1 + a_{45}X_1, \quad Y' = a_{44}Y_2 + a_{45}X_2, \quad \Phi_2^\circ = a_{44}\psi_0 + a_{45}\varphi_0 \qquad (2.8)$$

$$T_1^\circ = C_{11}\varepsilon_1^\circ + C_{12}\varepsilon_2^\circ + C_{16}\omega^\circ, \quad S^\circ = C_{16}\varepsilon_1^\circ + C_{26}\varepsilon_2^\circ + C_{66}\omega^\circ$$

$$T_2^\circ = C_{22}\varepsilon_2^\circ + C_{12}\varepsilon_1^\circ + C_{26}\omega^\circ, \quad C_{ik} = hB_{ik} \qquad (2.9)$$

Here a_{ik} – elastic constants; $T_i°$, $S°$ – internal tangential forces, C_{ik} – elongation hardness, $\varepsilon_i°$, $\omega°$ – the relative deformations of elongation and shear.

3. Using the geometric equations of the theory of elasticity, according to (2.6), (2.7) for the displacements of any point of the shell, with an accuracy of $1 \pm k_i^2h^2 \approx 1$ we obtain

$$u_\gamma = w + \gamma T^* + \gamma^2 M^* + \gamma^3 K^* + \gamma^4 N^* \tag{3.1}$$

$$\begin{aligned} u_\alpha = {} & u(1 + k_1\gamma) - \gamma \frac{1}{A}\frac{\partial w}{\partial \alpha} - \frac{\gamma^2}{2}(1 - {}^1/_3 k_1\gamma)\frac{1}{A}\frac{\partial T^*}{\partial \alpha} - \\ & - \frac{\gamma^3}{3}\left(1 - \frac{k_1\gamma}{2}\right)\frac{1}{A}\frac{\partial M^*}{\partial \alpha} + \gamma\left(1 + \frac{k_1\gamma}{2}\right)\frac{h^2}{8}\Phi_1° - \\ & - \frac{\gamma^3}{6}\left(1 + \frac{k_1\gamma}{4}\right)\Phi_1° + \gamma\left(1 + \frac{k_1\gamma}{2}\right)X^* + \frac{\gamma^2}{2h}\left(1 + \frac{k_1\gamma}{3}\right)X' \\ u_\beta = {} & v(1 + k_2\gamma) - \gamma \frac{1}{B}\frac{\partial w}{\partial \beta} - \frac{\gamma^2}{2}\left(1 - \frac{k_2\gamma}{3}\right)\frac{1}{B}\frac{\partial T^*}{\partial \beta} - \\ & - \frac{\gamma^3}{3}\left(1 - \frac{k_2\gamma}{2}\right)\frac{1}{B}\frac{\partial M^*}{\partial \beta} + \gamma\left(1 + \frac{k_2\gamma}{2}\right)\frac{h^2}{8}\Phi_2° - \\ & - \frac{\gamma^3}{6}\left(1 + \frac{k_2\gamma}{4}\right)\Phi_2 + \gamma\left(1 + \frac{k_2\gamma}{2}\right)Y^* + \frac{\gamma^2}{2h}\left(1 + \frac{k_2\gamma}{3}\right)Y' \end{aligned} \tag{3.2}$$

$u(\alpha, \beta)$, $v(\alpha, \beta)$, $w(\alpha, \beta)$ – the desired displacements of the middle surface of the shell

$$T^* = \frac{1}{h}T° - \frac{3a_{33}}{2h}K° + a_{33}Z_1, \quad M^* = \frac{6}{h^3}M° - \frac{h^2}{48}a_{33}Q° + \frac{a_{33}}{2h}Z_2 \qquad K^* = \frac{2a_{33}}{h^3}K°, \quad N^* = \frac{a_{33}}{24}Q° \tag{3.3}$$

$$T° = a_{13}T_1° + a_{23}T_2° + a_{36}S°, \quad M° = a_{13}M_1° + a_{23}M_2° + a_{36}H° \qquad Q° = \frac{1}{AB}[(B\varphi_0)_{,\alpha} + (A\psi_0)_{,\beta}], \quad K° = k_1M_1° + k_2M_2° \tag{3.4}$$

Examining formulas (3.1), (3.2) we note that, unlike all refined theories of the class in question, the geometric model of deformation of a shell here is such that all components of the displacement of some point of the shell depend nonlinearly on coordinate γ. In this case both all desired displacements $u(\alpha, \beta)$, $v(\alpha, \beta)$, $w(\alpha, \beta)$, as well as the known functions $T°(\alpha, \beta)$... $N^*(\alpha, \beta)$, which are defined according to the classical theory, will be functions only of curvilinear coordinates α and β.

Having the values of u_α, u_β, u_γ, with the help of the geometric córrelations of the three-dimensional theory of elasticity it is easy to determine the strain components e_α, e_β, $e_{\alpha\beta}$. The indicated deformations, on the strength of (3.1), (3.2), are represented in the form of polynomials in powers of γ, namely:

$$
\begin{aligned}
e_\alpha &= \varepsilon_1 + \gamma \varkappa_1{}^* + \gamma^2\eta_1 + \gamma^2\vartheta_1 \\
e_\beta &= \varepsilon_2 + \gamma \varkappa_2{}^* + \gamma^2\eta_2 + \gamma^3\vartheta_2 \\
e_{\alpha\beta} &= \omega + \gamma\tau^* + \gamma^2\nu + \gamma^3\lambda
\end{aligned}
\tag{3.5}
$$

The expansion coefficient $\varepsilon_1 \ldots \lambda$ not given here can be written by the usual method.

Examining the expansion ratios we note that, even if we are restricted to two members of expansions (3.5), we will not obtain the results of the classical theory, since the expansion ratios are basically distinguished from the appropriate coefficients of the classical theory. Here the coefficients $\kappa_i{}^*$, τ^* along with common members, which represent changes in curvature and the twisting of the middle surface of the shell, contain new elements of lateral deformations $e_{\alpha\gamma}$, $e_{\beta\gamma}$, e_γ.

4. Solving the equations of the generalized Hooke's law relative to the calculated stresses, according to (2.1), (3.5) we obtain

$$
\begin{aligned}
\sigma_\alpha &= B_{11}a_1 + B_{12}a_2 + B_{16}a' + \gamma(B_{11}b_1 + B_{12}b_2 + B_{16}b') + \\
&+ \gamma^2(B_{11}c_1 + B_{12}c_2 + B_{16}c') + \gamma^3(B_{11}d_1 + B_{12}d_2 + B_{16}d') \\
\sigma_\beta &= B_{22}a_2 + B_{12}a_1 + B_{26}a' + \gamma(B_{22}b_2 + B_{12}b_1 + B_{26}b') + \\
&+ \gamma^2(B_{22}c_2 + B_{12}c_1 + B_{26}c') + \gamma^3(B_{22}d_2 + B_{12}d_1 + B_{26}d') \\
\tau_{\alpha\beta} &= B_{16}a_1 + B_{26}a_2 + B_{66}a' + \gamma(B_{16}b_1 + B_{26}b_2 + B_{66}b') + \\
&+ \gamma^2(B_{16}c_1 + B_{26}c_2 + B_{66}c') + \gamma^3(B_{16}d_1 + B_{26}d_2 + B_{66}d')
\end{aligned}
\tag{4.1}
$$

where

$$
\begin{aligned}
B_{11} &= (a_{22}a_{66} - a_{26}^2)\Omega^{-1}, & B_{16} &= (a_{12}a_{26} - a_{22}a_{16})\,\Omega^{-1} \\
B_{22} &= (a_{11}a_{66} - a_{16}^2)\,\Omega^{-1}, & B_{26} &= (a_{12}a_{16} - a_{11}a_{26})\,\Omega^{-1} \\
B_{66} &= (a_{11}a_{22} - a_{12}^2)\,\Omega^{-1}, & B_{12} &= (a_{16}a_{26} - a_{12}a_{66})\,\Omega^{-1}
\end{aligned}
\qquad (4.2)
$$

$$
\begin{aligned}
&\Omega = (a_{11}a_{22} - a_{12}^2)\,a_{66} - 2a_{12}a_{16}a_{26} - a_{11}a_{26}^2 - a_{22}a_{16}^2 \\
&a_i = \varepsilon_i - a_{i3}\left(Z_1 - \frac{3}{2h}K^\circ\right), \qquad a' = \omega - a_{36}\left(Z_1 - \frac{3}{2h}K^\circ\right) \\
&b_i = \varkappa_i^* - a_{i3}\left(\frac{Z_2}{h} - \frac{h^2}{24}Q^\circ\right), \qquad b' = \tau^* - a_{36}\left(\frac{Z_2}{h} - \frac{h^2}{24}Q^\circ\right) \\
&c_i = \eta_i - a_{i3}\frac{6}{h^3}K^\circ, \qquad c' = \nu - a_{36}\frac{6}{h^3}K^\circ \\
&d_i = \vartheta_i - a_{i3}\frac{1}{6}Q^\circ, \qquad d' = \lambda - a_{36}\frac{1}{6}Q^\circ
\end{aligned}
\qquad (4.3)
$$

Thus, by formulas (4.1) we calculated stresses in the shell. These stresses change according to a nonlinear law through the shell. However, often in formulas (4.1) it is possible to be restricted to only the first two groups of terms, i.e., it is possible to assume a linear law of stress distribution σ_α, σ_β, $\tau_{\alpha\beta}$ through the shell.

The stresses σ_α, σ_β, $\tau_{\alpha\beta}$ have statically equivalent internal forces and moments, which will be defined by the formulas

$$
\begin{aligned}
T_1 &= C_{11}m_1 + C_{12}m_2 + C_{16}r + k_2\,(D_{11}n_1 + D_{12}n_2 + D_{16}s) \\
T_2 &= C_{22}m_2 + C_{12}m_1 + C_{26}r + k_1\,(D_{22}n_2 + D_{12}n_1 + D_{26}s) \\
S_1 &= C_{66}r + C_{16}m_1 + C_{26}m_2 + k_2\,(D_{66}s + D_{16}n_1 + D_{26}n_2) \\
S_2 &= C_{66}r + C_{16}m_1 + C_{26}m_2 + k_1\,(D_{66}s + D_{16}n_1 + D_{26}n_2)
\end{aligned}
\qquad (4.4)
$$

$$
\begin{aligned}
M_1 &= D_{11}n_1 + D_{12}n_2 + D_{16}s + k_2\,(D_{11}q_1 + D_{12}q_2 + D_{16}p) \\
M_2 &= D_{22}n_2 + D_{12}n_1 + D_{26}s + k_1\,(D_{22}q_2 + D_{12}q_1 + D_{26}p) \\
H_1 &= D_{66}s + D_{16}n_1 + D_{26}n_2 + k_2\,(D_{66}p + D_{16}q_1 + D_{26}q_2) \\
H_2 &= D_{66}s + D_{16}n_1 + D_{26}n_2 + k_1\,(D_{66}p + D_{16}q_1 + D_{26}q_2)
\end{aligned}
\qquad (4.5)
$$

where the following designations have been introduced:

$$
\begin{aligned}
m_i &= a_i + \frac{h^2}{12} c_i = e_i + \frac{h^2}{12} \eta_i - a_{i3} \left(Z_1 - \frac{1}{h} K^\circ \right) \\
r &= a' + \frac{h^2}{12} c' = \omega + \frac{h^2}{12} \nu - a_{36} \left(Z_1 - \frac{1}{h} K^\circ \right) \\
n_i &= b_i + \frac{3h^2}{20} d_i = \varkappa_i^* + \frac{3h^2}{20} \vartheta_i - a_{i3} \left(\frac{1}{h} Z_2 - \frac{h^2}{60} Q^\circ \right) \\
s &= b' + \frac{3h^2}{20} d' = \tau^* + \frac{3h^2}{20} \lambda - a_{36} \left(\frac{1}{h} Z_2 - \frac{h^2}{60} Q^\circ \right) \\
q_i &= a_i + \frac{3h^2}{20} c_i = e_i + \frac{3h^2}{20} \eta_i - a_{i3} \left(Z_1 - \frac{3}{5h} K^\circ \right) \\
p &= a' + \frac{3h^2}{20} c' = \omega + \frac{3h^2}{20} \nu - a_{36} \left(Z_1 - \frac{3}{5h} K^\circ \right)
\end{aligned}
\tag{4.6}
$$

5. Examining the above-derived formulas, equations and correlations we note that all calculation values of the shell are a function of three desired displacements u, v, w. Besides the desired ones they also contain certain new elements which, according to the given formulas, are determined from the solutions to the appropriate problem of a shell from the classical theory. (All these elements are disignated by "o.")

To get resolvent equations, to the derived ones we should add the equilibrium equations, the correlations of the continuity of deformations of the middle surface, and boundary conditions. These all differ in no way from the appropriate representations of the classical theory.

6. If for a shell the coefficients of the first quadratic form A, B and the curvatures of the middle surface k_1, k_2 are constant or, with sufficiently high accuracy, behave as constants, then with the accuracy of the technical theory of shells [1] we obtain the following resolvent equations relative to the desired displacements:

$$L_{11}(C_{ik})u + L_{12}(C_{ik})v + L_{13}(C_{ik})w = -X -$$
$$-\frac{3}{2}\left(P_{11}\frac{1}{A}\frac{\partial K^\circ}{\partial \alpha} + P_{66}\frac{1}{B}\frac{\partial K^\circ}{\partial \beta}\right) + h\left(P_{11}\frac{1}{A}\frac{\partial Z_1}{\partial \alpha} + P_{66}\frac{1}{B}\frac{\partial Z_1}{\partial \beta}\right) \tag{6.1}$$

$$L_{22}(C_{ik})v + L_{12}(C_{ik})u + L_{23}(C_{ik})w = -Y -$$
$$-\frac{3}{2}\left(P_{22}\frac{1}{B}\frac{\partial K^\circ}{\partial \beta} + P_{66}\frac{1}{A}\frac{\partial K^\circ}{\partial \alpha}\right) + h\left(P_{22}\frac{1}{B}\frac{\partial Z_1}{\partial \beta} + P_{66}\frac{1}{A}\frac{\partial Z_1}{\partial \alpha}\right) \tag{6.2}$$

$$L_{13}(C_{ik})u + L_{23}(C_{ik})v + L_{33}(D_{ik})w = Z - \frac{3}{2}(P_{11}k_1 + P_{22}k_2)K^\circ +$$
$$+\frac{h^2}{8}[E_1(D_{ik})\Phi_1^\circ + E_2(D_{ik})\Phi_2^\circ] + \frac{h^3}{288}P_1(P_{ik})Q^\circ + P_2(D_{ik}k_i)T^* -$$
$$-\frac{h^2}{12}P_1(P_{ik})Z_1 + h(P_{11}k_1 + P_{22}k_2)Z_1 + E_1(D_{ik})X^* + E_2(D_{ik})Y^* \tag{6.3}$$

where

$$\begin{aligned}
P_{11} &= B_{11}a_{13} + B_{12}a_{23} + B_{16}a_{36}\\
P_{22} &= B_{22}a_{23} + B_{12}a_{13} + B_{26}a_{36}\\
P_{66} &= B_{66}a_{33} + B_{16}a_{13} + B_{26}a_{23}
\end{aligned}$$

where for linear operators we have

$$E_1(B_{ik}) = B_{11}\frac{1}{A^3}\frac{\partial^3}{\partial \alpha^3} + 3B_{16}\frac{1}{A^2B}\frac{\partial^3}{\partial \alpha^2\partial \beta} +$$
$$+(B_{12} + 2B_{66})\frac{1}{AB^2}\frac{\partial^3}{\partial \alpha\partial \beta^2} + B_{26}\frac{1}{B^3}\frac{\partial^3}{\partial \beta^3}$$

$$E_2(B_{ik}) = B_{22}\frac{1}{B^3}\frac{\partial^3}{\partial \beta^3} + 3B_{26}\frac{1}{B^2A}\frac{\partial^3}{\partial \alpha\partial \beta^2} +$$
$$+(B_{12} + 2B_{66})\frac{1}{BA^2}\frac{\partial^3}{\partial \beta\partial \alpha^2} + B_{16}\frac{1}{A^3}\frac{\partial^3}{\partial \alpha^3}$$

$$L_{11}(C_{ik}) = \frac{C_{11}}{A^2}\frac{\partial^2}{\partial \alpha^2} + 2\frac{C_{16}}{AB}\frac{\partial^2}{\partial \alpha\partial \beta} + \frac{C_{66}}{B^2}\frac{\partial^2}{\partial \beta^2}$$

$$L_{22}(C_{ik}) = \frac{C_{22}}{B^2}\frac{\partial^2}{\partial \beta^2} + 2\frac{C_{26}}{AB}\frac{\partial^2}{\partial \alpha\partial \beta} + \frac{C_{66}}{A^2}\frac{\partial^2}{\partial \alpha^2}$$

$$L_{12}(C_{ik}) = \frac{C_{16}}{A^2}\frac{\partial^2}{\partial \alpha^2} + \frac{C_{12} + C_{66}}{AB}\frac{\partial^2}{\partial \alpha\partial \beta} + \frac{C_{26}}{B^2}\frac{\partial^2}{\partial \beta^2}$$

$$L_{13}(C_{ik}) = (k_1C_{11} + k_2C_{12})\frac{1}{A}\frac{\partial}{\partial \alpha} + (k_1C_{16} + k_2C_{26})\frac{1}{B}\frac{\partial}{\partial \beta}$$

$$L_{23}(C_{ik}) = (k_2C_{22} + k_1C_{12})\frac{1}{B}\frac{\partial}{\partial \beta} + (k_2C_{26} + k_1C_{16})\frac{1}{A}\frac{\partial}{\partial \alpha}$$

$$L_{33}(D_{ik}) = D_{11}\frac{1}{A^4}\frac{\partial^4}{\partial \alpha^4} + 4D_{16}\frac{1}{A^3B}\frac{\partial^4}{\partial \alpha^3\partial \beta} + 2(D_{12} +$$
$$+2D_{66})\frac{1}{A^2B^2}\frac{\partial^4}{\partial \alpha^2\partial \beta^2} + 4D_{26}\frac{1}{AB^3}\frac{\partial^4}{\partial \alpha\partial \beta^3} + D_{22}\frac{1}{B^4}\frac{\partial^4}{\partial \beta^4} +$$
$$+(k_1^2C_{11} + 2k_1k_2C_{12} + k_2^2C_{22})$$

$$P_1(P_{ik}) = P_{11}\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + 2P_{66}\frac{1}{AB}\frac{\partial^2}{\partial\alpha\,\partial\beta} + P_{22}\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2}$$

$$F_2(D_{ik}k_i) = (D_{11}k_1 + D_{12}k_2)\frac{1}{A^2}\frac{\partial^2}{\partial\alpha^2} + (D_{22}k_2 +$$
$$+ D_{12}k_1)\frac{1}{B^2}\frac{\partial^2}{\partial\beta^2} + 2(D_{16}k_1 + D_{26}k_2)\frac{1}{AB}\frac{\partial^2}{\partial\alpha\,\partial\beta}$$

The desired functions u, v, w (and thus all calculated values) should be determined from system of equations (6.1)-(6.3) whose left sides differ in no way from the left sides of the appropriate equations of the classical theory. As concerns the right parts of these equations, they are distinguished in principle from the right parts of the appropriate equations of the classical theory. Here, along with common load members X, Y, Z there are also certain reduced loads which are constructed with the help of the solutions to the appropriate problem of the classical theory.

7. Let us examine the problem of an axisymmetrically loaded orthotropic circular cylindrical shell, freely supported on its ends and supporting a normally applied load which changes across the surface of the shell according to the law $Z = Z^{+} = q \sin \pi\alpha/l$ (l – the length of the shell, q – the intensity of the load in the central section of the shell) (Fig. 2).

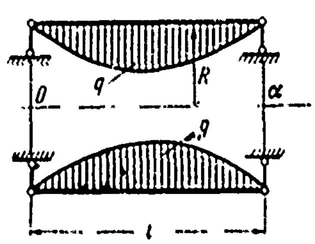

Fig. 2

For the desired functions $w = w(\alpha)$ and $F = F(\alpha)$, through which all calculated values of the problem are represented, we obtain

$$w = w_0\left[1 + A + \frac{hnl^2}{\pi^2 a_{22} R} B\right], \quad F = F_0\left[1 + A + \frac{\pi^2 B_{11} h^3 R}{12 l^2} B\right]$$

$$A = \frac{\pi^4 B_{11}^2 a_{22} a_{33} R^2 h^6}{96 l^6 [\ \ ]} + \frac{\pi^4 a_{22} B_{11} P_{11} R^2 h^4}{288 l^6 [\ \ ]} - \frac{\pi^2 a_{22} B_{12} h^2}{12 l^3 [\ \ ]} +$$

$$+ \frac{\pi^6 a_{22} a_{33} B_{12}^2 h^6}{96 l^6 [\ \ ]} - \frac{\pi^2 a_{33} B_{12} h^3}{24 l^3 R} + \frac{\pi^2 R_{11} h^2}{12 l^3}$$

$$B = -(a_{22} P_{22} + a_{12} P_{11})\left[\frac{\pi^4 a_{22} B_{12} h R}{8 l^4 [\ \ ]} - \frac{\pi^2}{2 l^2}\right]$$

$$[\ \ ] = 1 + \frac{\pi^4}{12} B_{11} a_{22} \frac{h^2 R^2}{l^4}$$

Let us examine a numerical example of a transversally isotropic shell [1]. The results of the calculation of values w/w_0 and F/F_0 at different values of the ratios $e = E/E'$ and $g = E/G'$, when $\nu = \nu' = 0.3$, $R/l = 1$, $h/l = 0.2$, are found in the table. In each block the upper numerals relate to w/w_0, and the lower – to F/F_0.

Table

e	$g = 0.0$	$g = 2.0$	$g = 5.0$	$g = 10.0$
0.0	1.0000 1.0000	1.0285 1.0285	1.0713 1.0713	1.1426 1.1426
1.0	0.9591 0.9949	0.9876 1.0234	1.0304 1.0662	1.1017 1.1375
2.0	0.9181 0.9898	0.9466 1.0183	0.9895 1.0611	1.0608 1.1324
5.0	0.7954 0.9746	0.8240 0.9969	0.8667 1.0459	0.9380 1.1172

Examining the table we note that with strong anisotropy, disregard of the phenomena associated with lateral deformations can lead to substantial errors. It is interesting to note that in certain cases the correction due to consideration of e_γ can exceed that due to consideration of transverse shears.